1. INTRODUCTION

Cheap measurement devices such as accelerometers, gyroscopes, or magnetometers are widely used in various navigation systems of ground and aerial vehicles in order to minimize the costs of measurement systems (Ahn and Won, 2009; Grewal and Andrews, 2008). In addition, they constitute one of the most important parts of unmanned aerial vehicles autopilots, as their reliability is crucial to control systems performance (Ali and Ullag Baig Mirza, 2010). With better knowledge about current position and orientation of unmanned aerial vehicle it is possible to perform more complicated, more precise and faster maneuvers (Chen and Xu, 2010). Therefore, the improvement of the quality of measurement systems is at almost all times desirable (Wagner and Kastl, 2004). Application of higher grade measurement devices can be far too expensive (Shojaei and Mohammad Shahri, 2011), while using the dose computation power does not increase the costs of the whole system (Haid and Breitenbach, 2004). Kalman filter is a practical computational algorithm which may be used as a tool for estimating unknown state vectors basing on noisy measurements (Simon, 2001).

Inertial measurement unit (IMU) senses three accelerations and three angular rates for different vehicles degrees of freedom (Titterton and Weston, 1997). Along with adequate computational algorithm IMU creates inertial navigation system (INS), which, however, gives biased and noisy position data, while assuring relatively high short term accuracy (Wendel et al., 2001; Sun et al., 2013). On the other hand, GPS receiver provides relatively less accurate position data, but it is not affected by the time drift. Hence, both of the aforementioned devices are mutually complementing one another (Hongwei et al., 2006). It is possible to integrate both sources of data in order to gain advantages and reduce or even eliminate disadvantages of separated measurements by means of Kalman filter (Mohamed and Schwarz, 1999). Apart from position, orientation is also determined by the use of three different data sources which are accelerometer, magnetometer characterized by long term accuracy and gyroscope characterized by short term accuracy. In this case, orientation may be estimated using Kalman filtering as well.

The STM32 platform has been used in order to minimize delay connected with computational effort required to compute matrices (Franca Junior and Morgado, 2010). Furthermore, it provided enough outputs and inputs to connect it with Inertial Measurement Unit, GPS receiver and computer.

2. KALMAN FILTER

Standard Kalman filter is a linear estimator, that has the ability to minimize error variance. Its equations are characterized by recursive type of computation, which may be relatively easily realized on a microprocessor. Kalman filter current output depends on current state and current inputs (Brookner, 1998). The filter is basing on linear systems equations:

\[ x_{k+1} = Ax_k + Bu_k + w_k \]  
\[ y_k = Hx_k + z_k \]

where \( A \), \( B \), and \( H \) are state, input and measurement matrices respectively, \( x \) is state of the system, \( u \) is known input to the system, \( y \) is the measured output, \( w \) is a process noise and \( z \) is a measurement noise. Vector \( x \) contains present state of the system, which can be estimated from given measurements in vector \( y \) and given input to the system determined in vector \( u \). However, we cannot entirely rely on information from \( y \) to obtain \( x \) vector, because \( y \) is noised (Bar-Shalom et al., 2001). Depending on the system, all of these quantities can be vectors (elements including more than one element).

Kalman filter assumes that process noise and measurement noise are not correlated with each other, whereas their average values are zero. In order to fulfill the aforementioned assumptions,
covariance matrices for process noise ($Q$), and measurement noise ($R$) are given by formulas (3) and (4).

$$Q = E(w_kw_k^T)$$  \hspace{1cm} (3)

$$R = E(z_kz_k^T)$$  \hspace{1cm} (4)

Kalman filter algorithm can be split up to two different stages called respectively time update and measurement update. During the first stage, values of vector $x$ (5) are being predicted and covariance matrix $P_k^{-}$ (6) is being computed.

$$x_k^- = Ax_{k-1} + Bu_{k-1}$$  \hspace{1cm} (5)

$$P_k^- = AP_{k-1}A^T + Q$$  \hspace{1cm} (6)

where $x_k^-$ is the predicted state estimation at actual time step, $P_k^-$ is the predicted estimated covariance matrix, $P_{k-1}$ is the updated estimated covariance at previous time step and $x_{k-1}$ is updated state estimation from previous time step.

Finally (updated) values are determined during the second stage of Kalman filter. The Kalman gains matrix is computed from the equation given as (7). The Riccati and Kalman filter equations are given by:

$$K_k = P_k^- H^T (HP_k^- H^T + R)^{-1}$$  \hspace{1cm} (7)

$$x_k = x_k^- + K_k (y_k - Hx_k^-)$$  \hspace{1cm} (8)

$$P_k = (I - K_k H) P_k^-$$  \hspace{1cm} (9)

where $K_k$ is the Kalman gain at actual time step, $x_k$ is the updated state estimation vector at actual time step, $y_k$ is the measurement vector at actual time step, $P_k$ is the updated estimated covariance matrix at actual time step, and $I$ is the identity matrix (Simon, 2001).

2.1. Orientation

In order to make Kalman filter applicable to parameters estimation of specific processes, dynamic system description is needed. In the case of orientation evaluation, which dynamic system is given by equations (10) and (11) if we denote $\alpha$ as angle, $\omega$ as angular rate, $q_b$ as gyroscope bias and $T$ as sample period.

$$\alpha_k = \alpha_{k-1} + (\omega_k - q_{bk-1})T$$  \hspace{1cm} (10)

$$q_{bk} = q_{bk-1}$$  \hspace{1cm} (11)

Gyroscope information can be distorted by bias error, which hence should be, and is estimated in state vector and included in aforementioned dynamic system equation, giving the algorithm the possibility of correcting angular velocity value.

Taking (12) as the state vector, matrices $A$, $B$, and $H$ are given respectively by (13), (14) and (15).

$$x = \begin{bmatrix} \alpha \\ q_b \end{bmatrix}$$  \hspace{1cm} (12)

$$A = \begin{bmatrix} 1 & -T \\ 0 & 1 \end{bmatrix}$$  \hspace{1cm} (13)

$$B = \begin{bmatrix} T \\ 0 \end{bmatrix}$$  \hspace{1cm} (14)

$$H = \begin{bmatrix} 1 & 0 \end{bmatrix}$$  \hspace{1cm} (15)

Input and measurements vectors are defined in (16) and (17) equations respectively. Input to the filter is the angular rate while the measurement is the angle.

$$u = [\omega]$$  \hspace{1cm} (16)

$$y = [\alpha]$$  \hspace{1cm} (17)

Matrix $Q$ representing system noise covariance and $R$ which is the measurement noise covariance matrix are as follows:

$$Q = \begin{bmatrix} T^2 \sigma^2_{\omega} & 0 \\ 0 & \sigma^2_{\omega} \end{bmatrix}$$  \hspace{1cm} (18)

$$R = [\sigma^2_{\omega}]$$  \hspace{1cm} (19)

where $\sigma_{\omega}$ represents orientation measurement noise and $\sigma_{\omega}$ represents angular rate measurement noise (Simon, 2001).

The use of three consecutive Kalman filter implementations for pitch, roll and yaw accordingly is adopted in order to reduce matrices size and therefore decrease computational requirements.

2.2. Position

In the case of position filtration, available information corresponds to current position and its second derivative – acceleration. Taking $S$ as travelled distance, $v$ as velocity, $a$ as acceleration, $a_b$ as accelerometer bias and $T$ as sample period, dynamic system used in the algorithm of Kalman filter, is described by three equations. The first equation (20) describes the way in which the position is being calculated. It uses values of previous position, previous velocity, current acceleration and previous accelerometer bias. The last mentioned value may, depending on the algorithm estimation accuracy, virtually precisely reduce related measurement error. The second equation (21) is used to compute velocity, having its previous value, an acceleration as well as an acceleration bias. The last equation (22) assigns value of accelerometer bias from previous time step to the current one.

$$S_k = S_{k-1} + v_{k-1}T + \frac{(a_k - a_{b,k-1})T^2}{2}$$  \hspace{1cm} (20)

$$v_k = v_{k-1} + (a_k - a_{b,k-1})T$$  \hspace{1cm} (21)

$$a_{b,k} = a_{b,k-1}$$  \hspace{1cm} (22)

The equations listed above may be represented in state space form, where three state variables are included in the state vector (23).

$$x = \begin{bmatrix} S \\ v \\ \alpha_b \end{bmatrix}$$  \hspace{1cm} (23)

Input and measurements vectors are defined in (24) and (25) equations respectively. Input to the filter is the acceleration while the measurement is the position.

$$u = [\alpha]$$  \hspace{1cm} (24)

$$y = [S]$$  \hspace{1cm} (25)

Matrices $A$, $B$ and $H$ describing the dynamic system are given in the following form:
Kalman Filter Realization for Orientation and Position Estimation on Dedicated Processor

\[
A = \begin{bmatrix}
1 & dt & -T^2 \\
0 & 1 & -2T \\
0 & 0 & 1
\end{bmatrix}
\]  
(26)

\[
B = \begin{bmatrix}
T^2 \\
2T \\
0
\end{bmatrix}
\]  
(27)

\[
H = [1 \ 0 \ 0]
\]  
(28)

Representations of system noise covariance and measurement noise covariance are adopted by \( Q \) (29) and \( R \) (30) matrices respectively.

\[
Q = \begin{bmatrix}
\frac{T^4}{4} \sigma^2_p & \frac{T^3}{2} \sigma^2_p & 0 \\
\frac{T^3}{2} \sigma^2_p & T^2 \sigma^2_p & 0 \\
0 & 0 & 0
\end{bmatrix}
\]  
(29)

\[
R = [\sigma_a]
\]  
(30)

where \( \sigma_p \) denotes position measurement noise and \( \sigma_a \) denotes acceleration measurement noise (Simon, 2001).

3. COMPLEMENTARY EQUATIONS

3.1. Rotation Matrix

Rotation matrix is used in order to transform values of measurements given in one frame to those expressed in a different one. Aforementioned matrix from the body frame to the navigation frame is given by:

\[
C^b_n = R_x(\psi)R_y(\theta)R_z(\phi)
\]  
(31)

where:

\[
R_x(\psi) = \begin{bmatrix}
\cos \psi & -\sin \psi & 0 \\
\sin \psi & \cos \psi & 0 \\
0 & 0 & 1
\end{bmatrix}
\]  
(32)

\[
R_y(\theta) = \begin{bmatrix}
\cos \theta & 0 & \sin \theta \\
0 & 1 & 0 \\
-\sin \theta & 0 & \cos \theta
\end{bmatrix}
\]  
(33)

\[
R_z(\phi) = \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \phi & -\sin \phi \\
0 & \sin \phi & \cos \phi
\end{bmatrix}
\]  
(34)

By substituting (32), (33) and (34) to (31):

\[
C^b_n = \begin{bmatrix}
\cos \phi \cos \psi + \sin \phi \sin \psi & \cos \phi \sin \psi - \sin \phi \cos \psi & \cos \phi \sin \theta \\
-\sin \phi \cos \psi + \cos \phi \sin \psi & \sin \phi \sin \psi + \cos \phi \cos \psi & -\sin \phi \cos \theta \\
-c \psi \sin \phi - s \psi \cos \phi - c \phi \cos \psi & c \phi \sin \psi + s \phi \sin \psi & c \phi \cos \psi + s \phi \cos \theta
\end{bmatrix}
\]  
(35)

where \( c \) represents cosine, \( s \) represents sine, \( \theta \) is a pitch angle, \( \phi \) is a roll angle and \( \psi \) is a yaw angle (Titterton and Weston, 1997).

3.2. Pitch and Roll Angles Derivation

By utilizing accelerometer data it is possible to compute coarse values of pitch (37) and roll (36) angles, which may further be used in filtration algorithm.

\[
\xi = -\tan^{-1} (\frac{a_y^b}{a_x^b})
\]  
(36)

\[
\eta = \tan^{-1} (\frac{a_z^b}{a_x^b})
\]  
(37)

where \( \xi \) is the derived roll angle, \( \eta \) is the derived pitch angle, \( a_y^b \) is the raw measured acceleration value in \( y \) axis of the body frame, \( a_x^b \) is the raw measured acceleration value in \( x \) axis of the body frame.

4. GPS ERRORS

Using GPS measurements may give rise to several error sources, such as: ionospheric error, tropospheric error, orbital error, satellite clock error, receiver clock error, multipath etc. Ionospheric and tropospheric errors are caused by slower signal propagation in comparison with vacuum speed of light. Satellite clock error and receiver clock error are strictly connected to inaccuracies of onboard clock, causing the offset of the satellite clock and the receiver clock accordingly with respect to the GPS-time. Orbital error is the error between true and estimated satellite positions. Multipath occurs when the signal is being reflected on the way from satellite to receiver instead of going straightforward (Romaniuk, 2013). Possible influence of those types of errors on the GPS measurement is presented in Tab. 1.

<table>
<thead>
<tr>
<th>Error type</th>
<th>Error value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ionospheric error</td>
<td>2-20m</td>
</tr>
<tr>
<td>Tropospheric error</td>
<td>0.5-5m</td>
</tr>
<tr>
<td>Orbital error</td>
<td>0-3m</td>
</tr>
<tr>
<td>Satellite clock error</td>
<td>0-3m</td>
</tr>
<tr>
<td>Receiver clock error</td>
<td>0-10m</td>
</tr>
<tr>
<td>Multipath</td>
<td>0-5m</td>
</tr>
<tr>
<td>Noise (others)</td>
<td>0.25-0.5m</td>
</tr>
</tbody>
</table>

5. EXPERIMENTAL SETUP

Mobile research platform has been designed on the base of STM32F103 microcontroller. Chosen element is characterized by relatively low cost, numerous communication peripheries and adequate running frequency. Moreover, it can be programmed in C/C++ programming language and it runs at 72 MHz frequency (http://www.kamami.pl). Measurements have been taken by means of Razor 9 DOF inertial measurement unit (IMU) and GPS receiver with MTK MT3339 chipset.

Single axis gyroscope (LY530ALH), two axis gyroscope (LPR530AL), triple axis accelerometer (ADXL335), triple axis magnetometer (HMC5843) are the components of the used IMU. Linear accelerations and angular rates measured in aforesmen-
tioned inertial system are transmitted by means of UART interface. Razor 9 DOF runs at 50Hz sampling rate. Sensors specifications of gyroscopes (Tab. 2.), accelerometer (Tab. 3.) and magnetometer (Tab. 4.) are relatively low. Since there are two devices used to measure angular rate, two columns for each of them are included in Tab. 1. In the case of accelerometer device, it was needed to split X, Y and Z axis in order to fully describe the aforementioned sensor.

GPS receiver is using UART interface in order to transmit NMEA packets to STM32F103 microcontroller. Those packets include different types of information regarding not solely position, but satellites status, time and different parameters as well (http://www.gpsinformation.org/dale/nmea.htm). In the presented application, configuration of GPS receiver defined actualization frequency of position measurement at 1 Hz (one time per second).

### Tab. 2. Gyroscopes specifications (https://www.sparkfun.com/)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>LYS30ALH</th>
<th>LPR530AL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measurement range [°/s]</td>
<td>300</td>
<td>300</td>
</tr>
<tr>
<td>Sensitivity [mV/°/s]</td>
<td>3.33</td>
<td>3.33</td>
</tr>
<tr>
<td>Nonlinearity [% FS]</td>
<td>±1</td>
<td>±1</td>
</tr>
<tr>
<td>Bandwidth [Hz]</td>
<td>140</td>
<td>140</td>
</tr>
<tr>
<td>Rate noise density [°/s/√Hz]</td>
<td>0.035</td>
<td>0.035</td>
</tr>
</tbody>
</table>

### Tab. 3. Accelerometer specifications (https://www.sparkfun.com/)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>ADXL335</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measurement range X, Y, Z [g]</td>
<td>±3.6</td>
</tr>
<tr>
<td>Sensitivity X, Y, Z [mV/g]</td>
<td>300</td>
</tr>
<tr>
<td>Nonlinearity X, Y, Z [%]</td>
<td>±0.3</td>
</tr>
<tr>
<td>Bandwidth X, Y [Hz]</td>
<td>1600</td>
</tr>
<tr>
<td>Bandwidth Z [Hz]</td>
<td>550</td>
</tr>
<tr>
<td>Noise Density X, Y [μg/√Hz]</td>
<td>150</td>
</tr>
<tr>
<td>Noise Density Z [μg/√Hz]</td>
<td>300</td>
</tr>
</tbody>
</table>

### Tab. 4. Magnetometer specifications (https://www.sparkfun.com/)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>HMC5843</th>
</tr>
</thead>
<tbody>
<tr>
<td>Field range [gauss]</td>
<td>±4</td>
</tr>
<tr>
<td>Linearity [% ± FS]</td>
<td>&lt;0.1</td>
</tr>
<tr>
<td>Bandwidth [kHz]</td>
<td>10</td>
</tr>
<tr>
<td>Resolution [milli − gauss]</td>
<td>7</td>
</tr>
<tr>
<td>Signal-to noise ratio [dB]</td>
<td>&gt;70</td>
</tr>
</tbody>
</table>

Employed GPS receiver has different configurable values of actualization frequency, however the lowest one rated at 1Hz has been used in order to have better comparison of position interpolation. Constraints of maximum altitude, velocity and acceleration values are defined for ordinary GPS receivers which may be utilized in civil area, in order to prevent military applications such as cruise missile systems (Pace, 1996; Rush, 2000; http://www.armscontrol.org/documents/mtcr). Characteristics of used GPS receiver are presented in Tab. 5., which includes most significant ones such as position accuracy, velocity accuracy, actualization frequency and aforementioned civilian GPS receivers limitations.

### Tab. 5. GPS receiver specifications (http://www.aliexpress.com/)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>MTK MT3339</th>
</tr>
</thead>
<tbody>
<tr>
<td>Receiver</td>
<td>L1 frequency, C/A code, 66 channels</td>
</tr>
<tr>
<td>GPS position accuracy</td>
<td>&lt;3.0 m 50% CEP</td>
</tr>
<tr>
<td>GPS velocity accuracy [m/s]</td>
<td>0.1</td>
</tr>
<tr>
<td>Actualization frequency [Hz]</td>
<td>1-10</td>
</tr>
<tr>
<td>Sensitivity [dBm]</td>
<td>-165</td>
</tr>
<tr>
<td>Maximum altitude [m]</td>
<td>18 000</td>
</tr>
<tr>
<td>Maximum velocity [m/s]</td>
<td>515</td>
</tr>
<tr>
<td>Maximum acceleration [g]</td>
<td>4</td>
</tr>
</tbody>
</table>

The first step needed to estimate values of orientation angles is to prepare data acquired from accelerometer, gyroscope and magnetometer. The aforesaid data preparation involves digitalization, transformation and range adjustment. Subsequently, acquired data is evaluated in Kalman filter, where estimated values of pitch, roll and yaw angles come from. The process mentioned above can be seen in Fig. 1. Measured values from gyroscope are used during the time update stage of Kalman filter algorithm and are entered in the u vector. Moreover, the evaluated angles, basing on the data from accelerometer and magnetometer are used in the measurement update stage, and are inputted to the algorithm in the y vector. The angles derived from accelerometer and magnetometer data are used to verify the data from the time update stage, and to make sufficient corrections if necessary.

![Fig. 1. Information flow for orientation estimation (Romaniuk, 2013)](image1)

![Fig. 2. Information flow for position estimation (Romaniuk, 2013)](image2)
Position estimation scheme (Fig. 2), on the other hand, is somewhat different. While acceleration and GPS position are given in various units, it is necessary to transform one into another or vice versa. Furthermore, values from inertial measurement unit and GPS receiver are given in different reference frames, which has to be adjusted by means of adequate rotation matrix (35). Obviously, it is needed to have attitude information before doing such transformations. Hence, estimation of pitch, roll and yaw values is done first, before computing position values. Subsequently calculated information of acceleration in navigation frame and GPS data is used to estimate position by Kalman filter. Information that comes from inertial measurement unit is used during the time update and is entered as \( v \) vector, while the data from GPS, inputted in \( y \) vector are used during the measurement update of Kalman filter algorithm. GPS data are used for comparison with the values achieved during the time update and optionally to correct them.

All the data processing related to the Kalman filtration algorithm has been performed on STM32 microcontroller. Calculations were performed on matrixes, hence it was needed to implement functions for matrix operations such as addition, subtraction, multiplication, and inversion. Kalman filter algorithm has been running on 50 Hz rate, which was the same as the actualization frequency of inertial measurement unit. Utilizing USB HID communication availability, calculated output has been saved on the laptop, where results were processed.

Research has been performed on the open terrain, characterized by relatively small amount of physical obstacles, thus providing good GPS satellites visibility. Starting point of the test route is covering the ending one. Data have been recorded during the movement along the specified test route, however because of the time needed to get GPS position fix, it has been recorded for longer term than alone run lasted.

6. RESULTS

When performing many tests on the same route, it was observed that GPS information had many drawbacks. There were unexpected satellite signal losses causing high accuracy deterioration, which was not necessarily induced by physical obstacles. Furthermore, used GPS receiver seemed to have needed relatively long time in order to start and get the first position fix.

Some signal losses immunity is achieved by using Kalman filter which integrates data from GPS device and the inertial measurement unit. In the case where the rapid position change occurs on the GPS receiver outage, the corresponding change on the output of Kalman filter will not be similarly fast if the inertial measurement unit data do not coincide with GPS change. On the other hand, this behavior depends on the reliability of IMU. Poor quality of inertial measurement unit devices greatly reduces capabilities of position estimation when shortage of GPS signal occurs.

While conducting the orientation algorithm performance evaluation, comparison between the unfiltered and filtered data has been made, where the former represents angles derived from raw magnetometer data and the latter one is the output of the Kalman filter. On the other hand, in the case of position estimation algorithm, first data set refers to raw, unaided, unfiltered GPS data, while the other one shows performance of the Kalman filter algorithm.

As illustrated in Fig. 3, usage of Kalman filter gave relatively high noise reduction in comparison with raw IMU data, what is shown by unstable measurement trend. Furthermore, what is also important, relatively high output dynamics are preserved. This is evidenced in high dynamic movements shown at 10-13 s, 14-18 s and 19-27 s for pitch, roll and yaw angles respectively.
7. CONCLUSIONS

A solution of INS/GPS systems integration in real time has been presented in the following paper. Orientation filtering algorithm is characterized by rewarding measurement noise reduction and resolution increase, which can be seen on pitch, roll and yaw graphs. On the other hand in the case of the position integration algorithm, the main advantage is about achieving higher update frequency rated at about 50Hz, which in comparison with 1Hz, characteristic to used GPS receiver, is great enhancement. Unfortunately, the results show rather poor Kalman filter altitude estimation. This is principally caused by low GPS accuracy in this area, and should be further improved by using better source for measuring height. Further increasing the frequency, Kalman filter algorithm is running at, would decrease the time step length (Lee and Saloic, 1997).

It is possible to improve quality of measurement system by introducing model which would estimate a larger number of sensor errors (Han and Wang, 2012). Knowledge of specified sensor errors helps in accurate position estimation in case of loss of GPS signal. It is caused by the fact that, through the application of precise corrections, relatively accurate determination of measurement signals is becoming possible. As the instance of the system that introduces such model, deeply integrated INS/GPS system may be given (Luo et al., 2012).

Further improvement is possible by adopting higher grade of inertial measurement unit and/or GPS receiver. Sensors characterized by reliable specifications are more expensive, and, as a result, not always possible to use in low cost systems. Introducing more measurement devices for measuring the same quantity of measurement parameter is another way of navigation system quality enhancement (Caron et al. 2006). Proposed solution may be further studied in order to improve overall performance of the measurement system (Gosiewski and Ortyl, 1999; Gibbs, 2011; Ning and Fang, 2007).

REFERENCES